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Temperature fluctuation spectrum in the dissipation range for statistically isotropic turbulent flow

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In the present paper we obtain a theoretical expression for the temperature fluctuation spectrum for a Prandtl number of approximately one and for the region where both viscosity and molecular heat conductivity are important. An asymptotic theory for very large wavenumbers of the temperature spectrum in the turbulent flow is developed. The assumption of smallness of the correlation coefficient between the product of small-scale components of velocities at two points and the corresponding product of small-scale components of temperatures is used. The results of simultaneous measurements of streamwise velocity fluctuations and temperature fluctuations carried out in the plane of symmetry of a two-dimensional wake behind a slightly heated cylinder ($R_\lambda = 270$) in a wind tunnel is consistent with this assumption.

The main result of the theory developed is the appearance of a bump in the temperature spectrum for a Prandtl number of approximately one.

1. Introduction

It is well known that for very large Reynolds numbers there exist universal spectra for velocity and passive-scalar fields in turbulent flow for the so-called equilibrium range of spectra (we neglect here the intermittency effects and consider our results to be for fixed values of velocity and scalar dissipation rate; see, for example, Monin & Yaglom 1975). This range is characterized by parameters $\{\epsilon, \nu, \chi, N\}$. Here, ϵ is the rate of energy dissipation per unit of mass; ν is the viscosity; χ is the molecular coefficient of heat conductivity (or the molecular diffusion coefficient if the scalar is molecular density); and N is the rate of dissipation of mean-squared temperature fluctuations, i.e. $N \equiv \chi\langle(\nabla T)^2\rangle$, where angle brackets denote an ensemble average.

The general expression for the one-dimensional spatial temperature spectrum $V_T(k)$ obtained from dimensional analysis is given by the formula (see, for example, Monin & Yaglom 1975)

$$V_T(k) = N\epsilon^{-\frac{1}{3}}k^{-\frac{5}{3}}\phi_T(k\eta, Pr) \quad \text{for} \quad kL \gg 1.$$

Here k is one component of the wave vector, $Pr = \nu/\chi$ is the Prandtl number,

$$\eta = (\nu^3/\epsilon)^{\frac{1}{4}}$$

is the Kolmogorov scale of turbulence, L is the outer scale of turbulence, and ϕ_T is a universal dimensionless function depending on dimensionless arguments. The Kolmogorov scale characterizes the inner scale of the velocity field.

We also use the Corrsin scale:

$$\eta_C = (\chi^3/\epsilon)^{\frac{1}{4}} = \eta/Pr^{\frac{3}{4}}.$$

For $Pr \leqslant 1$, the Corrsin scale characterizes the range of the temperature spectrum in which the molecular heat conductivity is important. For $Pr \geqslant 1$, Batchelor's scale, η_B, is the scale for which both viscous and molecular heat conductivity are effective:

$$\eta_B = (\nu\chi^2/\epsilon)^{\frac{1}{4}} = \eta/Pr^{\frac{1}{2}}.$$

Using scales L, η, η_C, and η_B we can classify the possible ranges of the temperature spectrum (see Batchelor 1959):

the inertial–convective range

$$1/L \ll k \ll \min(1/\eta, 1/\eta_C),$$

the viscous–diffusive range

$$k \gg \max(1/\eta, 1/\eta_B),$$

the inertial–diffusive range

$$1/\eta_C \ll k \ll 1/\eta \quad \text{for} \quad Pr \ll 1,$$

the viscous–convective range

$$1/\eta \ll k \ll 1/\eta_B \quad \text{for} \quad Pr \gg 1.$$

In fact, the boundaries between these ranges are more accurately given by $0.1/\eta$, $0.1/\eta_C$, and $0.1/\eta_B$ respectively.

For $Pr \gg 1$ or $Pr \ll 1$ only three of these four ranges exist. For $Pr \approx 1$ only the inertial–convective and viscous–diffusive ranges exist.

In the inertial–convective range the parameters ν and χ are not important, and for this case the following well-known formula is valid:

$$V_T(k) = MC_T^2\, k^{-\frac{5}{3}} = B_T N\epsilon^{-\frac{1}{3}}k^{-\frac{5}{3}}, \quad C_T^2 = a^2 N/\epsilon^{\frac{1}{3}}, \quad Ma^2 = B_T.$$

Here a^2 and B_T are dimensionless constants (B_T is known as an Oboukhov–Corrsin constant), and $M = \Gamma(\frac{5}{3})\sin(\frac{1}{3}\pi)/(2\pi) = 0.1244$. This formula was first obtained by Oboukhov (1949) in terms of structure functions (Oboukhov's $\frac{2}{3}$ law for temperature fluctuations):

$$D_T(r) \equiv \langle (T_1 - T_2)^2 \rangle = C_T^2\, r^{\frac{2}{3}}.$$

The quantity $N \equiv \chi\langle(\nabla T')^2\rangle = -\frac{1}{2}(\partial/\partial t)\langle T'^2\rangle$ as a rate of dissipation of temperature fluctuations was first introduced by Oboukhov (1949). The same formula was obtained in a somewhat different way by Yaglom (1949). A bit later the formula for $V_T(k)$ was independently obtained by Corrsin (1951) (Corrsin's $-\frac{5}{3}$ law, which is the spectral equivalent of the $\frac{2}{3}$ law).

A common description of both the viscous–convective and viscous–diffusive ranges was proposed by Batchelor (1959). Using some simple natural physical assumptions about the interaction between small-scale velocity field components (in the viscous range of its spectrum) and the temperature field, he obtained, for $\nu \gg \chi$ ($Pr \gg 1$) for the viscous–convective range $1/\eta \ll k \ll 1/\eta_B$,

$$V_T(k) = qN\tau_0\, k^{-1} \quad \text{for} \quad 1/\eta \ll k \ll 1/\eta_B.$$

Here $\tau_0 = (\nu/\epsilon)^{\frac{1}{2}}$ is the Kolmogorov inner scale for time and q is a dimensionless constant. Batchelor's first estimate of q was $q \approx 2$. The constant q in fact is a free parameter in Batchelor's theory. It determines the intersection point k_* of the $-\frac{5}{3}$ and -1 asymptotic solutions:

$$k_* \eta = (B_T/q)^{\frac{3}{2}}.$$

Grant *et al.* (1968) estimated the value of q as $q = 3.9 \pm 1.5$, for $k_* \eta = 0.024 \pm 0.008$ ($Pr = 9.4$, water); Gibson, Lyon & Hirschsohn (1970) obtained a similar estimate for $k_* \eta = 0.035 \pm 0.005$ ($Pr = 700$).

Note that the $-\frac{5}{3}$ and -1 laws have been experimentally verified with high precision for different Prandtl or Schmidt numbers†.

If we look at experimental data for air ($Pr = 0.72$) (see, for example, Hill's 1978*a* review), we see that between the inertial–convective and the viscous–diffusive ranges there exists a 'bump'. Some optical and underwater propagation features caused by the bump have been analysed by Gurvich, Kallistratova & Martvel (1977), Hill & Clifford (1978), Hill (1978*b*), and Elliot, Kerr & Pincus (1979).

At first sight, this bump resembles Batchelor's range of the spectrum, but it exists for $Pr \approx 1$, which is not described by his theory. It is impossible to explain this bump on the basis of asymptotic theories, which use the limit $Pr \rightarrow \infty$ or $Pr \rightarrow 0$ (such as Gibson's 1968 theory, which is based on similarity analysis and because of this is an asymptotical theory with respect to the Prandtl number).

Different phenomenological theories have been used to explain the bump (Hill 1978*a*, *c*, 1980). They all used speculative models of interactions between different spectral components of temperature fluctuations. For example, the equation

$$\partial F(k)/\partial k = -2\chi k^2 \Gamma(k)$$

was used, where $\Gamma(k)$ is the three-dimensional temperature spectrum multiplied by $4\pi k^2$, and $\partial F(k)/\partial k = -T(k)$ is an unknown spectral transfer function, connected with the third mixed moment of velocity and temperature. Different functions (analytical expressions) for $F(k)$ were proposed, which are compatible with the $-\frac{5}{3}$ and -1 asymptotical solutions and contain several numerical parameters. It is possible to choose these functions and parameters such that the $\Gamma(k)$ obtained describes the experimental data very well. But from an ideological point of view it is the same as choosing a good analytical approximation directly for $\Gamma(k)$. If we have such an approximation for $\Gamma(k)$, we can obtain $F(k)$ from the same equation.

In the present paper our goal is to obtain a theoretical expression for the temperature fluctuation spectrum for $Pr \approx 1$ (for air it is approximately 0.72). In this simplest case only the inertial–convective and viscous–diffusive ranges exist.

To obtain the general form of an asymptotic solution of the temperature spectrum for the high-frequency spatial components, we use the equation for the high-frequency (space–time) temperature correlation function, which is similar to the Chandrasekhar equation for the velocity field. In the case of the velocity field, similar assumptions make it possible to obtain good agreement between the theoretical and the experimental results in the viscous range of the spectrum (see Dubovikov & Tatarskii 1987).

After matching this asymptotic solution for the temperature spectrum with the $-\frac{5}{3}$

† Note that the numerical simulation of Lesieur & Rogallo (1989) for $Pr \approx 1$ leads to the k^{-1} behaviour of the spectrum for small-scale temperature components (for the range between the outer scale and the $-\frac{5}{3}$ low range). We do not know of any experimental evidence for similar behaviour of the temperature spectrum in this range.

law we will obtain the composite asymptotic solution for the whole range. In the case of temperature fluctuations for $Pr \approx 1$ it leads to the bump in the spectrum.

The theoretical results in this paper are due to V. Tatarskii and M. Dubovikov, the experimental results of §3 are due to A. Praskovsky and M. Karyakin.

2. Basic equation

We start from the heat transport equation in a given incompressible turbulent velocity field:

$$\left(\frac{\partial}{\partial t}-\chi\nabla^2\right)T = Q-\nabla\cdot(\boldsymbol{v}T), \qquad \frac{\partial v_q}{\partial x_q}=0 \tag{2.1}$$

(where summation is over repeated indices). Here Q represents the heat sources, which are concentrated in the region of small wavenumbers and small frequency. Let us introduce the Fourier transforms

$$T(\boldsymbol{k},\omega) = (2\pi)^{-4}\int \exp\left[-\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)\right] T(\boldsymbol{r},t)\,\mathrm{d}^3\boldsymbol{r}\,\mathrm{d}t,$$

$$Q(\boldsymbol{k},\omega) = (2\pi)^{-4}\int \exp\left[-\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)\right] Q(\boldsymbol{r},t)\,\mathrm{d}^3\boldsymbol{r}\,\mathrm{d}t$$

and suppose that

$$Q(\boldsymbol{k},\omega)\equiv 0 \quad \text{if} \quad |\boldsymbol{k}|>k_0, \quad \omega>\omega_0.$$

Let us also introduce a function $S(\boldsymbol{r},t)$ such that its Fourier transform,

$$S(\boldsymbol{k},\omega) = (2\pi)^{-4}\int \exp\left[-\mathrm{i}(\boldsymbol{k}\boldsymbol{r}-\omega t)\right] S(\boldsymbol{r},t)\,\mathrm{d}^3\boldsymbol{r}\,\mathrm{d}t,$$

is equal to zero for the region $|\boldsymbol{k}|<k_0, |\omega|<\omega_0$ and equal to one for $|\boldsymbol{k}|\gg k_0, |\omega|\gg\omega_0$. In this case

$$S(\boldsymbol{k},\omega)\,Q(\boldsymbol{k},\omega)\equiv 0.$$

Then if we multiply the heat transport equation by $S(\boldsymbol{r}-\boldsymbol{r}', t-t')$ and integrate over $\boldsymbol{r}, t$, the term

$$\int Q(\boldsymbol{r},t)\,S(\boldsymbol{r}-\boldsymbol{r}',t-t')\,\mathrm{d}\boldsymbol{r}\,\mathrm{d}t\equiv 0$$

vanishes, because the convolution of the functions corresponds to the product of the spectra and $S(\boldsymbol{k},\omega)\,Q(\boldsymbol{k},\omega)\equiv 0$.

Let

$$[T(\boldsymbol{r},t)]'\equiv\int S(\boldsymbol{r}-\boldsymbol{r}',t-t')\,T(\boldsymbol{r}',t')\,\mathrm{d}\boldsymbol{r}'\,\mathrm{d}t'$$

denote the small-scale component of the temperature field. We also use the $[\,]'$ notation to denote the small-scale components of the other functions. We call this operation filtering. The filtered heat transport equation takes the form

$$\partial[T]'/\partial t-\chi\Delta[T]' = -[\boldsymbol{v}(\boldsymbol{r},t)\cdot\boldsymbol{\nabla}T(\boldsymbol{r},t)]'$$

(because $\bar{Q}=0$). Now we write the same equation for the other $(\boldsymbol{r},t)$ point, multiply these two equations, and average the product:

$$(\partial/\partial t_1-\chi\Delta_1)(\partial/\partial t_2-\chi\Delta_2)\langle[T_1]'[T_2]'\rangle = \langle[v_j(1)\,\partial T/\partial x_{j1}]'\,[v_n(2)\,\partial T/\partial x_{n2}]'\rangle. \tag{2.2}$$

We denote $\langle [T_1]' [T_2]' \rangle \equiv \bar{B}_T$ the high-frequency correlation function, and assume that the high-frequency components of T are statistically homogeneous and stationary. In this case $B_T = B_T(\boldsymbol{\rho}, t)$, where $\boldsymbol{\rho} = \boldsymbol{r}_1 - \boldsymbol{r}_2$, $t = t_1 - t_2$, and

$$\frac{\partial}{\partial t_1} = \frac{\partial}{\partial t}, \quad \frac{\partial}{\partial t_2} = -\frac{\partial}{\partial t}, \quad \frac{\partial}{\partial \boldsymbol{r}_1} = \frac{\partial}{\partial \boldsymbol{\rho}}, \quad \frac{\partial}{\partial \boldsymbol{r}_2} = -\frac{\partial}{\partial \boldsymbol{\rho}}, \quad \left(\frac{\partial}{\partial t_1} - \chi \Delta_1\right)\left(\frac{\partial}{\partial t_2} - \chi \Delta_2\right) = \chi^2 \Delta^2 - \frac{\partial^2}{\partial t^2}.$$

The equation obtained takes the form

$$\left(\chi^2 \Delta^2 - \frac{\partial^2}{\partial t^2}\right) B_T(\boldsymbol{\rho}, t) = \left\langle \left[v_j(\boldsymbol{r}_1, t_1) \frac{\partial T(\boldsymbol{r}_1, t_1)}{\partial x_{j1}} \right]' \left[v_n(\boldsymbol{r}_2, t_2) \frac{\partial T(\boldsymbol{r}_2, t_2)}{\partial x_{n2}} \right]' \right\rangle. \tag{2.3}$$

For statistically homogeneous and statistically stationary turbulence this equation is exact, but not closed.

If $\boldsymbol{v}$ and T are considered Gaussian and statistically isotropic, the exact relation

$$\left\langle v_j(\boldsymbol{r}_1, t_1) \frac{\partial T(\boldsymbol{r}_1, t_1)}{\partial x_{j1}} v_n(\boldsymbol{r}_2, t_2) \frac{\partial T(\boldsymbol{r}_2, t_2)}{\partial x_{n2}} \right\rangle = \langle v_j(\boldsymbol{r}_1, t_1) v_n(\boldsymbol{r}_2, t_2) \rangle \left\langle \frac{\partial T(\boldsymbol{r}_1, t_1)}{\partial x_{j1}} \frac{\partial T(\boldsymbol{r}_2, t_2)}{\partial x_{n2}} \right\rangle$$

is valid. The deviations from this relation are caused by non-Gaussian terms, and they are proportional to the third- and higher-order cumulants.

The main assumption used in this paper is that it is possible to use the same relation as an approximation but only for the small-scale components of turbulence. (It is known, see O'Brien & Francis 1963, that this assumption leads to negative values in the large-scale range of the spectrum. In our case, i.e. small-scale components, we do not have this difficulty.) It is also known (see Antonia *et al.* 1984) that the tails of the probability distribution for the difference between the temperatures at two spatial points are essentially non-Gaussian. This is very important for calculations of high-order structure functions, but since we use this approximation for the second moment only we can expect that the relative error of this relation will not be too big.

Section 3 describes the experiment carried out to verify this assumption. The measurements do not contradict the assumption, with an accuracy of about 10%.

Thus, if we suppose that the relation

$$\left\langle \left[v_j(\boldsymbol{r}_1, t_1) \frac{\partial T(\boldsymbol{r}_1, t_1)}{\partial x_{j1}} \right]' \left[v_n(\boldsymbol{r}_2, t_2) \frac{\partial T(\boldsymbol{r}_2, t_2)}{\partial x_{n2}} \right]' \right\rangle$$
$$= \langle [v_j(\boldsymbol{r}_1, t_1)]' \, [v_n(\boldsymbol{r}_2, t_2)]' \rangle \left\langle \left[\frac{\partial T(\boldsymbol{r}_1, t_1)}{\partial x_{j1}} \right]' \left[\frac{\partial T(\boldsymbol{r}_2, t_2)}{\partial x_{n2}} \right]' \right\rangle \tag{2.4}$$

is valid, (2.3) takes the form

$$\left(\chi^2 \Delta^2 - \frac{\partial^2}{\partial t^2}\right) B_T(\boldsymbol{\rho}, t) = -B_{jn}(\boldsymbol{\rho}, t) \frac{\partial^2}{\partial \rho_j \partial \rho_n} B_T(\boldsymbol{\rho}, t). \tag{2.5}$$

Here $B_{jn}(\boldsymbol{\rho}, t)$ is the correlation tensor for the small-scale components of the velocity field. For the isotropic case we can represent $B_{jn}(\boldsymbol{\rho}, t)$ in terms of the longitudinal correlation function B_{LL}, resulting in the equation

$$\frac{\partial^2 B_T}{\partial t^2} - \frac{\alpha^2}{\rho^4} \frac{\partial}{\partial \rho}\left(\rho^4 \frac{\partial^3 B_T}{\partial \rho^3}\right) = \frac{1}{\rho^2} \frac{\partial}{\partial \rho}\left[\rho^2 B_{LL}(\rho, \tau) \frac{\partial B_T}{\partial \rho}\right]. \tag{2.6}$$

Here B_{LL} is the filtered longitudinal velocity correlation function, and dimensionless variables

$$\tau = (\epsilon/\nu)^{\frac{1}{2}}(t_1 - t_2), \quad \rho = |\boldsymbol{r}_1 - \boldsymbol{r}_2|/\eta$$

are used. We use the notation α for the inverse Prandtl number:

$$\alpha = \chi/\nu = (Pr)^{-1}.$$

The estimates based on the diagram technique (similar to the technique of Dubovikov & Tatarskii 1987) show that deviations from (2.4) increase for $Pr \gg 1$. Thus we can suppose that (2.4) is valid for $Pr \approx 1$ only.

A similar closure idea was used by O'Brien & Francis (1963) to investigate the evolution of the scalar spectrum in the decay process: they obtained negative values for the spectrum in the large-scale range. Our approach uses a similar quasi-Gaussian approximation for small-scale (space–time) components only, and (2.6) describes not decay, but stationary small-scale turbulence.

A similar approach for the velocity field (Dubovikov & Tatarskii 1987) led to good agreement with experimental data in the viscous range of the turbulent spectrum. Because of that result, we attempted to use a similar approach for the temperature field.

3. Experimental verification of a closure hypothesis

The aim of this section is to present experimental data for the quantities on both sides of (2.4) and to examine the relative accuracy of the equation.

Many experimental papers contain joint measurements of velocity and temperature fluctuations, but we could not find results in the literature directly concerning the quantities in (2.4). The results in some papers (Antonia & Van Atta 1975; Antonia & Chambers 1980; Park 1976) are useful for estimating the accuracy of this relationship.

We present in this section some preliminary estimates of the following quantities (where u is the streamwise component of velocity fluctuations, T is the temperature fluctuation, and T_x is the streamwise derivative of temperature):

$$\frac{\langle u(x)\, u(x+r)\, T(x)\, T(x+r)\rangle}{\langle u(x)\, u(x+r)\rangle \langle T(x)\, T(x+r)\rangle} - 1 = \beta(r), \tag{3.1}$$

$$\frac{\langle u(x)\, u(x+r)\, T_x(x)\, T_x(x+r)\rangle}{\langle u(x)\, u(x+r)\rangle \langle T_x(x)\, T_x(x+r)\rangle} - 1 = \gamma(r), \tag{3.2}$$

$$\frac{\langle u(x)\, T_x(x+r)\rangle \langle u(x+r)\, T_x(x)\rangle}{\langle u(x)\, u(x+r)\rangle \langle T_x(x)\, T_x(x+r)\rangle} = \rho(r). \tag{3.3}$$

All the quantities in (3.1)–(3.3) were measured in frequency bands ranging from a low-cutoff frequency f_{min} up to high cutoff frequency f_c for different values of f_{min}.

We can expect on the basis of results of Antonia & Van Atta (1975) and Antonia & Chambers (1980) that $\beta(r)$ and $\gamma(r)$ will decrease with decreasing r and become very small for $r \approx \eta$. If that is so, it means that the hypothesis that $\gamma = 0$ to achieve closure in the equation for the temperature second moment in the viscous–diffusive range does not contradict experimental data.

The experiment was carried out in a low-turbulence direct-flow wind tunnel. The flow behind a vertical, electrically heated cylinder (length 350 mm, diameter 35 mm) was analysed. The measurements were carried out in the tunnel cross-section at a distance $x/d = 42.9$ behind the cylinder. The velocity of the external flow was 13 m/s; the Reynolds number was 3×10^4.

We used two platinum Wollaston 0.63 μm wires with active length $b = 0.25$ mm. The distance between wires was $h = 0.8$ mm. One wire was used as a hot-wire anemometer operating at constant temperature (the relative heating was 1.6); the other one was used as a resistance thermometer. The hot-wire anemometer output signal, $u(t)$, and the thermometer output signals $T(t)$ and $T_t(t)\,(T_t(t) = \partial T/\partial t)$ were passed through the low-pass filters (high cutoff frequency $f_c = 8$ kHz at 36 dB/octave). To obtain the time derivative $T_t(t)$, we used an analogous differentiator with linear characteristics in the band up to 16 kHz. The r.m.s. signal-to-noise ratio levels in the plane of symmetry of the wake was 270 for velocity fluctuations, 43 for temperature, and 8 for $T_t(t)$.

To obtain the spatial characteristics from our temporal measurements we used the Taylor frozen-turbulence hypothesis. To estimate the mean value of the energy dissipation rate $\langle \epsilon \rangle$ and the scalar dissipation $\langle N \rangle$, we used the local isotropy hypotheses, i.e.

$$\langle \epsilon \rangle = 15\nu \langle (\partial u/\partial x)^2 \rangle, \quad \langle N \rangle = 3\chi \langle (\partial T/\partial x)^2 \rangle.$$

The integral scale L, Taylor's microscale λ, and Kolmogorov's scale η were estimated by

$$L = \langle u^2 \rangle^{-1} \int_0^\infty \langle u(x)\, u(x+r) \rangle \,\mathrm{d}r, \quad \lambda = \left\{ \frac{\langle u^2 \rangle}{\langle (\partial u/\partial x)^2 \rangle} \right\}^{\frac{1}{2}}, \quad \eta = \left(\frac{\nu^3}{\langle \epsilon \rangle} \right)^{\frac{1}{4}}.$$

All the results presented in this section were obtained at a single point: in the plane of symmetry of the wake behind the cylinder. The mean values of energy dissipation and scalar dissipation rate were $\langle \epsilon \rangle = 17.2\ \mathrm{m^2/s^3}$ and $\langle N \rangle = 0.266\ \mathrm{K^2/s}$. The turbulence scales were $L = 55.1$ mm, $\eta = 0.12$ mm (i.e. $L/\eta = 460$), and $\lambda = 3.83$ mm. The Reynolds number was $R_\lambda = \langle v^2 \rangle^{\frac{1}{2}} \lambda/\nu = 270$. The spatial resolution of the measurements was $b/\eta = 2.1$ for independent velocity or temperature measurements.

An inertial range of about one decade was obtained in our experiment: $0.005 < k\eta < 0.1$, which is natural for $R_\lambda \approx 10^2$. Note that to get a high R_λ value, measurements were carried out at a relatively small distance from the cylinder ($x = 42.9d$), i.e. in a not fully developed part of the wake. Thus the scale η is small and the ratio h/η is relatively big. But we consider only the inertial range $r > 10\eta$ where the spatial averaging $h/\nu = 6.7$ is not so important.

We note also that near the cylinder the fluctuation structure is determined by the shedding of regular vortices (see, for example, Sarpkaya 1979). But these regular vortices correspond to small wavenumbers, and therefore they do not seem to be important for flow fluctuations in the region of high wavenumbers. We did not observe any peaks in the spectra of velocity and temperature at $k\eta \approx 3.8 \times 10^{-2}$, which corresponds to the Strouhal number of vortex shedding, $S = fd/U_0 \approx 0.16$.

Measurements of $\beta(r)$, $\gamma(r)$, and $\rho(r)$ were carried out for three different values of the low cutoff frequency, $f_{\min}$: 0 Hz (without filtering), 150 Hz ($V/2\pi f_{\min}\eta = 106$), and 1500 Hz ($V/2\pi f_{\min}\eta = 10.6$). Values of $\langle u(x)\, u(x+r) \rangle$ and $\langle T(x)\, T(x+r) \rangle$, and of the fourth moment $\langle u(x)\, u(x+r)\, T(x)\, T(x+r) \rangle$ are different for different values of $f_{\min}$, but normalized values of $\beta(r)$, $\gamma(r)$, and $\rho(r)$ are essentially similar. For example, the measured values of these quantities are presented in table 1, for $r = 15\eta$. For $r/\eta > 30$, the difference between the values of $\beta(r)$ for these three values of $f_{\min}$ is less than 1%. Taking into account that there is no essential difference in the measured

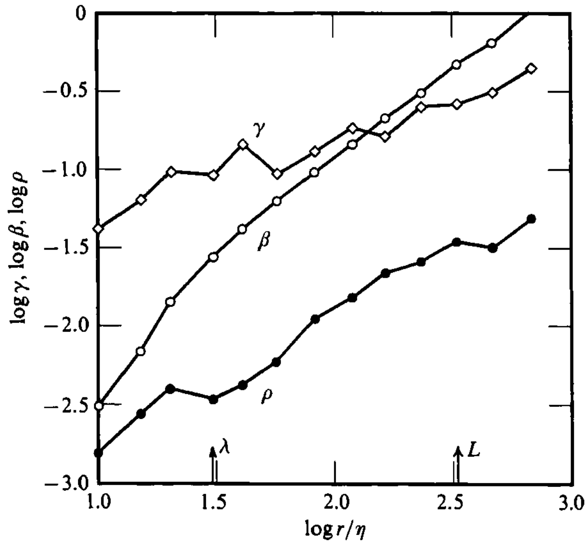

FIGURE 1. The results of measurements of $\beta(r)$, $\gamma(r)$, and $\rho(r)$. The arrows mark the outer scale and Taylor microscale of turbulence.

	$f_{min} = 0$ Hz	$f_{min} = 1500$ Hz
$\langle u(x)\, u(x+r)\rangle$ (m^2/s^2)	1.00	0.391
$\langle T(x)\, T(x+r)\rangle$ (°C^2)	0.105	0.0408
$\langle u(x)\, u(x+r)\, T(x)\, T(x+r)\rangle$ ($K^2\, m^2/s^2$)	0.106	0.0161
$\log \beta(r)$	−2.17	−2.14

TABLE 1. Correlation values for two different cutoff frequencies

values of $\beta(r)$, $\gamma(r)$, and $\rho(r)$ for different f_{min}, we present below the results for $f_{min} = 0$ only.

Figure 1 presents the results of measurements of $\beta(r)$, $\gamma(r)$, and $\rho(r)$. We cannot estimate their values for $r \leqslant h$, but it is obvious that all of these quantities decrease if r decreases, and the rate of decrease for $\beta(r)$ and $\gamma(r)$ is greater for $r \leqslant \lambda$ than for $r \geqslant \lambda$. For $r = 10\eta$ the measured results are $\beta = 2 \times 10^{-3}$, $\gamma = 4 \times 10^{-2}$, which does not contradict hypothesis (2.4).

Unfortunately, the relatively large distance between the wires in the experiment makes it impossible to verify this hypothesis in the viscous–diffusive range for $r < 10\eta$. Also, the Reynolds number was not large enough. Therefore it would be useful to plan additional measurements with Reynolds numbers $R_\lambda > 10^3$ and with spatial resolution $b < \eta, h < \eta$.

A priori, it was possible to obtain a value for γ of about unity, and in that case hypothesis (2.4) and all the conclusions based on it make no sense. On the other hand, experimental results obtained cannot be considered as evidence of the validity of (2.4). We can consider the result obtained as only a first estimate of the value of γ, and this estimate does not contradict the hypothesis used.

4. Investigation of singularities of the temperature correlation function in the complex ρ-plane

Our next problem is to investigate the asymptote of the spatial temperature spectrum in the region of high wavenumbers. We investigate this spectrum in the viscous region of the velocity spectrum, which was obtained by Dubovikov & Tatarskii (1987). In Batchelor's (1959) terms it is the viscous–diffusive range.

It is known that the asymptotic form for the spectrum for large k is determined by the singular point of the correlation function nearest to the real axis. Thus we begin now to investigate the singularities of $\bar{B}_T(\rho, \tau)$ on the complex ρ-plane.

The function $\bar{B}_T(\rho, \tau)$ obeys the differential equation (2.6). It is known that all the singular points of the solution of this equation coincide with the singular points of the equation coefficients.

The coefficients of (2.6) are singular at two points: the first is $\rho = 0$; the second is the point where the function $\bar{B}_{LL}(\rho, \tau)$ is singular. In our previous paper (Dubovikov & Tatarskii 1987) we found that the function $\bar{B}_{LL}(\rho, \tau)$ has a singularity at the point $\rho = \mathrm{i}\rho_0(\tau)$, which is nearest to the real axes.

First we investigate the singularity at $\rho = 0$. The most singular term in the expansion of $\bar{B}_T(\rho, \tau)$ near this point is in the form

$$\bar{B}_T(\rho, \tau) = A(\tau)/\rho^p.$$

If we substitute this term into (2.6) and take into account that function $\bar{B}_{LL}(\rho, \tau)$ is regular at this point, we obtain $p = 1$. This means that the corresponding term in the one-dimensional Fourier transform has the form $\mathrm{Const} \times A(\tau) \ln k$. But for finite Prandtl number ($\chi \neq 0$) the integral of the one-dimensional spectrum over the large-k region is finite, and therefore we must set Const $= 0$. Thus, the singularity at $\rho = 0$ is not essential for the asymptotic form of the spectrum.

Now we investigate the second singularity, at $\rho = \mathrm{i}\rho_0(\tau)$. Dubovikov & Tatarskii (1987) showed that the behaviour of $\bar{B}_{LL}(\rho, \tau)$ near this point has the form

$$\bar{B}_{LL}(\rho, \tau) = -30/[\rho - \mathrm{i}\rho_0(\tau)]^2 - 120\mathrm{i}/\{13\rho_0(\tau)[\rho - \mathrm{i}\rho_0(\tau)]\} + \dots \quad (4.1)$$

According to the general theory of differential equations, we seek a solution for $\bar{B}_T(\rho, \tau)$ near the singular point $\rho = \mathrm{i}\rho_0(\tau)$ in the form

$$\bar{B}_T(\rho, t) = [\rho - \mathrm{i}\rho_0(\tau)]^{-\mu}\{1 + \mathrm{i}a_1(\tau)[\rho - \mathrm{i}\rho_0(\tau)] + \dots\}. \quad (4.2)$$

If we substitute expansions (4.1) and (4.2) into (2.6) and compare the coefficients of the same powers of $[\rho - \mathrm{i}\rho_0(\tau)]$, we obtain (after rather extended algebra) the following relations:

$$\left.\begin{aligned} \alpha^2(\mu+1)(\mu+2) &= 30, \\ a_1(\tau) &= \mu(9-2\mu)/[13(\mu-1)\,\rho_0(\tau)]. \end{aligned}\right\} \quad (4.3)$$

Note that the term $\partial^2 B_T/\partial\tau^2$ in (2.6) has no effect on the quantities μ and $a_1(\tau)$ because it is connected with the singularities of lower order. This means that the time shift τ appears in μ and $a_1(\tau)$ only as a parameter (no derivatives with respect to τ arise). We can set this parameter to zero. It corresponds to consideration of a pure spatial temperature correlation function and spectrum. We obtained the same result (Dubovikov & Tatarskii 1967) for the function $\bar{B}_{LL}(\rho, \tau)$ near its singular point $\rho = \mathrm{i}\rho_0(\tau)$. The parameter $\rho_0(0) = \xi$ was estimated as

$$\xi = \rho_0(0) \approx 10.85. \quad (4.4)$$

This estimate is rather stable and is in good agreement with the experimental data.

On the basis of (4.3) we obtain, for the positive root μ,

$$\mu = \tfrac{1}{2}\{[1+120(Pr)^2]^{\frac{1}{2}}-3\}. \qquad (4.5a)$$

It is clear from (4.5a) that a positive root exists only for

$$Pr \geqslant (15)^{-\frac{1}{2}} \approx 0.258.$$

If this condition is not fulfilled, we must choose the negative root

$$\mu' = -\tfrac{1}{2}\{[1+120(Pr)^2]^{\frac{1}{2}}+3\}, \qquad (4.5b)$$

which corresponds to the branch point with a positive exponent.

After substituting (4.4) into (4.3) we obtain for the coefficient $a_1 = a_1(0)$

$$a_1 = \mu(9-2\mu)/[13\xi(\mu-1)]. \qquad (4.6)$$

Thus, the behaviour of the function $B(\rho) = \bar{B}_T(\rho, 0)$ in the neighbourhood of the singular point $\rho = \mathrm{i}\rho_0(0) = \mathrm{i}\xi$ is given by the formula

$$B(\rho) = (\rho-\mathrm{i}\xi)^{-\mu} + \mathrm{i}a_1(\rho-\mathrm{i}\xi)^{-\mu+1}+\dots. \qquad (4.7)$$

5. One-dimensional asymptotic form of the temperature spectrum for the viscous–diffusive range

We consider here the one-dimensional spatial temperature spectrum because this function is usually measured in direct experiments. We use the following definition of the spectrum:

$$V_T(k) = (2\pi)^{-1}\int_{-\infty}^{\infty} \exp(\mathrm{i}k\rho)B(\rho)\,\mathrm{d}\rho. \qquad (5.1)$$

(All the definitions and notation coincide with those used in Tatarskii 1971.) If we substitute the expansion (4.7) into (5.1) and calculate integrals (it is also possible to use Watson's lemma), we obtain the asymptotic expansion for the spectrum for large k:

$$V_T(k) = [\Gamma(\mu)]^{-1}\cos(\tfrac{3}{2}\pi\mu)\exp(-k\xi)\,k^{\mu-1}[1+(\mu-1)\,a_1\,k^{-1}+\dots].$$

Note that (3.5) is linear with respect to $B_T(\rho, \tau)$, so its spectrum $V_T(k)$ contains some unknown factors. These factors must be determined from additional information or from the experimental data. We denote this coefficient by D_1, and using (4.6) for a_1, we obtain for $V_T(k)$ the following expansion:

$$V_T(k) = D_1\exp(-k\xi)\,k^{\mu-1}\{1+\mu(9-2\mu)\,(13\xi)^{-1}k^{-1}+\dots\}. \qquad (5.2)$$

Now we must estimate the matching point between this asymptotic formula and the $-\frac{5}{3}$ law. For this estimate we use the well-known relation (in which the variables have natural dimensions)

$$4\int_0^{\infty} k^2 V_T(k)\,\mathrm{d}k = \frac{2N}{3\chi}. \qquad (5.3)$$

It is well known that in the inertial–convective range the functions $D(r)$ and $V_T(k)$ have the form (using the dimensional variables)

$$D(r) = C_T^2 r^{\frac{2}{3}}, \quad V_T(k) = MC_T^2\,k^{-\frac{5}{3}},$$

where $C_T^2 = a^2N/\epsilon^{\frac{1}{3}}$, $M = \Gamma(\frac{5}{3})\,3^{\frac{1}{2}}/(4\pi) = 0.1244\dots$, and a^2 is a numerical constant.

In general it is possible to represent $V_T(k)$ in the form

$$V_T(k) = MC_T^2 k^{-\frac{5}{3}} \phi_1(k\eta), \tag{5.4}$$

where the subscript 1 in $\phi_1(k\eta)$ denotes that it is a one-dimensional spectrum.

It is clear that $\phi_1(k\eta) \approx 1$ for $k\eta \ll 1$, but it must have a form for $k\eta \gg 1$ such that (5.2) is valid. If we return to the dimensionless variables, we obtain the following asymptotic formulae for $\phi_1(k)$:

$$\phi_1(k) = \begin{cases} 1 & \text{for} \quad k \ll 1 \\ D \exp(-k\xi)\, k^{\mu+\frac{2}{3}}[1+\mu(9-2\mu)(13\xi)^{-1}k^{-1}+\ldots] & \text{for} \quad k \gg 1. \end{cases} \tag{5.5a, b}$$

Here D is a new constant, which is the ratio of the constants in (5.2) and (5.4).

We assume that it is possible to choose, for these two asymptotic expressions, a matching point κ such that for $k < \kappa$ (5.5 a) is valid, but for $k > \kappa$ (5.5b) is applicable. If we introduce the step function

$$\theta(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ 1 & \text{for} \quad x > 0, \end{cases}$$

it is possible to rewrite $\phi_1(k)$ in the following form:

$$\phi_1(k) = \theta(\kappa - k) + \theta(k-\kappa) D \exp(-k\xi)\, k^{\mu+\frac{2}{3}}[1+\mu(9-2\mu)(13\xi)^{-1}\kappa^{-1}+\ldots]. \tag{5.5c}$$

We next wish to choose the point κ. We first substitute (5.4) into (5.3). Again using the dimensionless variables, we obtain the equality

$$\int_0^\infty k^{\frac{1}{3}} \phi_1(k)\,\mathrm{d}k = \frac{1}{6\alpha Ma^2}. \tag{5.6}$$

If we use (5.5c) in (5.6), we get after integration

$$\tfrac{3}{4}\kappa^{\frac{4}{3}} + D\xi^{-(\mu+2)}[\Gamma(\mu+2, \kappa\xi) + \tfrac{1}{13}\mu(9-2\mu)\,\Gamma(\mu+1,\kappa\xi)+\ldots] = 1/(6\alpha Ma^2). \tag{5.7}$$

Here
$$\Gamma(\alpha, x) = \int_x^\infty t^{\alpha-1}\exp(-t)\,\mathrm{d}t \tag{5.8}$$

is the non-complete gamma function.

Equation (5.7) connects the two unknown constants D and κ. A second equation we can use is the spectrum continuity condition at the point κ. It is the same as the continuity condition for the function $\phi_1(k)$:

$$1 = D\exp(-\kappa\xi)\,\kappa^{\mu+\frac{2}{3}}[1+\mu(9-2\mu)(13\xi)^{-1}\kappa^{-1}+\ldots]. \tag{5.9}$$

We can consider (5.7) and (5.9) as a system of transcendental algebraic equations for the unknown constants D and κ. This system of equations can be analysed numerically. A similar approach was used by Gibson (1968) and Dubovikov & Tatarskii (1987).

The important parameters of the system are the Prandtl number Pr and, depending on it, constants α, μ; the constant a^2 in the $\frac{2}{3}$ temperature law; and the number ξ, which characterizes the velocity spectrum in the viscous range.

First, we note that the system of equations analysed does not have a solution for all values of the parameters. The numerical constant ξ was determined by Dubovikov & Tatarskii (1987) with good accuracy, and its value of 10.85 was used for further calculations and did not vary. The main calculations were made for air, which is characterized by the parameters $Pr = 0.72, \alpha = 1.39$, $\mu = 2.48$.

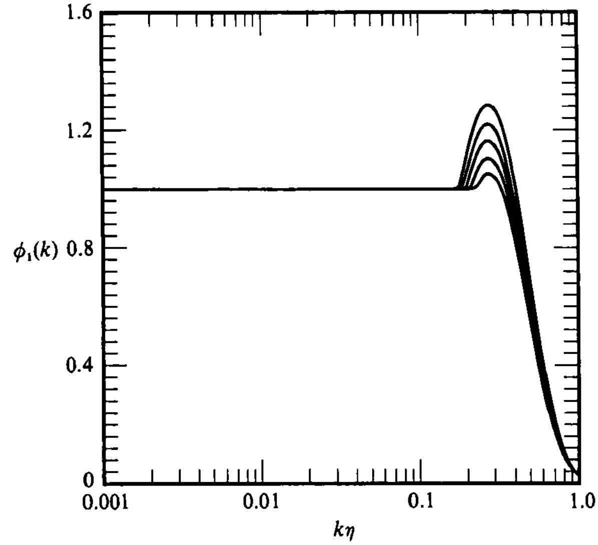

FIGURE 2. A set of curves illustrating the dependence of the function $\phi_1(k)$ on the value of a^2 for fixed $Pr = 0.72$ and $\xi = 10.85$. The upper curve corresponds to $a^2 = 2.4$, the lower one to $a^2 = 2.8$, intermediate values are 2.5, 2.6, 2.7.

For the values of these constants considered, a solution of the system of equations exists only for $a^2 < 2.8$. Note that this critical value of a^2 belongs to the range of a^2 values possible from the experimental point of view.

For $a^2 < 2.8$ in the region $k > \kappa$ on the curve $\phi_1 = \phi_1(k)$, the spectral bump appears. The bump has been obtained in many experiments: Champagne *et al.* (1977), Williams & Paulson (1977), and Time (1981).

One of the first estimates of a^2 (Tatarskii 1956) was $a^2 = 5.6$, but many of the more recent measurements indicate a large range of values: from 1.1 to 9 (see Yaglom 1981). To obtain good agreement between the experimental values for the bump and theory, it is necessary to choose the value $a^2 = 2.3$ (see §6 of this paper). This value belongs to the range of observed values of a^2.

The bump's value and position also depend on the Prandtl number. If the Prandtl number increases, the bump in the spectrum moves to the region of large wavenumbers. For example, the position of the bump's maximum is $k_0 = 0.10$ for $Pr = 0.4$, $k_0 = 0.27$ for $Pr = 0.72$ and $k_0 = 0.42$ for $Pr = 1$.

In figure 2 the curves presented illustrate the dependence of the function $\phi_1(k)$ on the value of a^2 for fixed μ and ξ. Using these curves, we can choose the best value of a^2 from the point of view of experimental values for the bump.

6. Comparison of asymptotic theory results with experimental data

Various experimental data on the temperature fluctuation spectra in the viscous–diffusive range have been obtained in wind tunnels, in the atmosphere, and in the ocean (see Champagne *et al.* 1977; Williams & Paulson 1977; Time 1981; and Hill's 1978*a* review). Some of these measurements were made by different, small, low-inertial devices; others were made by non-direct methods (see Tatarskii 1971) based on measurements of the spectra of fluctuations of light intensity and on the angle of wavefront arrival. Note that Gurvich *et al.* (1974) gave the first indications of the bump's existence, but their data were not accurate enough to affirm it.

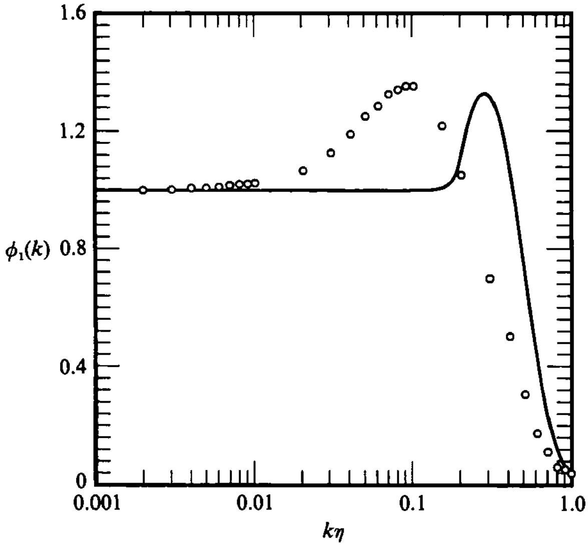

FIGURE 3. The function $\phi_1(k)$, calculated according to (5.5*c*), for the parameters $\xi = 10.85$, $Pr = 0.72$, $a^2 = 2.3$, $D = 1323$, $\kappa = 0.1715$. The maximum of $\phi_1(k)$ is located at the point $k_0 = 0.274$ and $\phi_1(k_0) = 1.351$. The data points are experimental values from Champagne *et al.* (1977), normalized according to the condition $\phi_1(0) = 1$. The maximum of the experimental curve is at $k_0 = 0.1$ and $\phi_1(k_0) = 1.34$.

The function $\phi_1(k)$, calculated according to (5.5), is presented in figure 3 for the following values of the parameters: $\xi = 10.85$, $Pr = 0.73$, $a^2 = 2.3$. The values of parameters D and κ, obtained from the solution of the system of equations (5.7) and (5.9), were $D = 1323$, $\kappa = 0.1715$. The maximum of $\phi_1(k)$ is at the point $k_0 = 0.274$ and $\phi_1(k_0) = 1.351$.

The experimental data, presented on the same plot, were taken from Champagne *et al.*'s (1977) paper and normalized according to the condition $\phi_1(0) = 1$. These data are typical for such measurements. The $\phi_1(k_0)$ value of the maximum on the experimental curve is 1.34 and coincides with the result from calculations (due to the choice of $a^2 = 2.3$). But for the experimental points the position of the maximum is $k_0 = 0.1$; i.e. it is in the region of the largest scale rather than at the maximum of the theoretical curve.

The most likely reason for the difference in the bump position for the theoretical result compared with the experimental data is inadequate accuracy of the asymptotic expansion (5.2), in which we used only two terms. To elucidate this, we consider the value $k = k_{\max}$, which corresponds to the maximum of ϕ_1. It is given by

$$k^{(2)}_{\max} = \{13 + 3\mu(\mu+2) + [9\mu^4 - 198\mu^3 + 1245\mu^2 - 195\mu + 169]^{\frac{1}{2}}\}/(39\xi). \tag{6.1}$$

The superscript (2) means that we used the two-term formula (5.2) for the temperature spectrum and the corresponding two-term formula (5.5) for ϕ_1. But if we neglect the second term in (5.6),

$$[1 + \mu(9-2\mu)(13\xi)^{-1}k^{-1} + \ldots],$$

we get the corresponding one-term formula for $k^{(1)}_{\max}$:

$$k^{(1)}_{\max} = [\mu + \tfrac{2}{3}]/\xi. \tag{6.2}$$

For a given Pr, (6.2) leads to bigger values of $k_{\max}$ than does (6.1). For example,

$k^{(2)}_{\max} = 0.27$ and $k^{(1)}_{\max} = 0.29$ for $Pr = 0.72$ and $\xi = 10.85$. This means that refining the asymptotic expansion from the one-term formula to the two-term one shifts the maximum position in the correct direction.

The real expansion parameter used is $(k\xi)^{-1}$, and it is about unity in the region of the bump. It is well known that in this case it is rather difficult to improve the results of the asymptotic expansion by means of increasing the number of terms. This might explain the quantitative difference between the theory presented and the experimental data. But at the same time it seems that the accuracy of the basic equation (2.6) is much greater than the accuracy of its approximate solution (the asymptotic expansion).

7. Conclusion

An asymptotic theory is developed for the temperature field in turbulent flow with very large Reynolds numbers and Prandtl number of about one, with the goal of describing the viscous–diffusive range of the spectrum.

The basic assumption of our approach is that the correlation coefficient for the product of small-scale components of velocity and small-scale temperature fluctuations, for a pair of points separated by a distance on the order of the Kolmogorov scale, is small. This assumption about the mixed fourth-moment behaviour is not contradicted by the results of our experiments in a wind tunnel. The result of Dubovikov & Tatarskii (1987) for the asymptotic form of the velocity spectrum in the viscous range is used.

The theoretical results show that a bump appears between the inertial–convective and viscous–diffusive ranges of the spectrum for $Pr \approx 1$, as observed in many experiments (it is necessary to distinguish this bump from the one that appears according to Batchelor's k^{-1} law for $Pr \gg 1$).

Hill's (1978*a*) review compares experimental results with several phenomenological theories. In these theories different formulae are proposed to describe the mixed third moment. Using these formulae, it is possible to close the second-moment temperature equation and to solve it. The resulting expression depends on the functions and the free parameter chosen for the closure approximation. It is possible to choose them in such a way that the agreement between the experimental results and the results of calculations is quite satisfactory in a wide range of spatial frequencies, including the bump.

As we mentioned in §1, this approach is equivalent to the direct approximation of the experimental data by means of analytical expressions. We do not know of any other approach that is able to describe the spectral behaviour, including the appearance of the bump, in the viscous–diffusive range when the Prandtl number is approximately one.

In contrast to the models reviewed by Hill (1978*a*), the method developed here does not use arbitrary suppositions about the spectral transfer function, but is based on a simple closure model that does not introduce any free parameters to the solution. It leads naturally to the appearance of the bump in the temperature spectrum.

The quantitative disagreement between the theory and experimental data for the position of the bump is not very great, and is more likely to be due to the inaccuracy of the asymptotic expansion (5.2) than the inaccuracy of the basic equation (2.6). This means that it is possible to improve the results by using some other methods to solve this equation.

We cannot, also, exclude the influence of intermittency. As noted by Keller & Yaglom (1970), these effects are very important in the extreme short-wave region $k\eta \gg 1$ of turbulence.

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REFERENCES

ANTONIA, R. A. & CHAMBERS, A. J. 1980 On the correlation between turbulent velocity and temperature derivatives in the atmospheric surface layer. *Boundary-Layer Met.* **18**, 399–410.

ANTONIA, R. A., HOPFINGER, E. J., GAGNE, Y. & ANSELMET, F. 1984 Temperature structure functions in turbulent shear flows. *Phys. Rev.* A **30**, 2704–2707.

ANTONIA, R. A. & VAN ATTA, C. W. 1975 On the correlation between temperature and velocity dissipation fields in a heated turbulent jet. *J. Fluid Mech.* **67**, 273–288.

BATCHELOR, G. K. 1959 Small-scale variation of convected quantities like temperature in turbulent fluid. Part 1. General discussion and the case of small conductivity. *J. Fluid Mech.* **5**, 113–133.

CHAMPAGNE, F. H., FRIEHE, C. A., LARUE, J. C. & WYNGAARD, J. C. 1977 Flux measurements, flux-estimation techniques, and fine-scale turbulence measurements in the unstable surface layer over land. *J. Atmos. Sci.* **34**, 515–530.

CORRSIN, S. 1951 On the spectrum of isotropic temperature fluctuations in isotropic turbulence. *J. Appl. Phys.* **22**, 469–473.

DUBOVIKOV, M. M. & TATARSKII, V. I. 1987 Calculation of the asymptotic form of the spectrum of locally isotropic turbulence in the viscous range. *Sov. Phys. JETP* **66**, 1136–1141.

ELLIOT, R. A., KERR, J. R. & PINCUS, P. A. 1979 Optical propagation in laboratory-generated turbulence. *Appl. Opt.* **18**, 3315–3323.

GIBSON, C. H. 1968 Fine structure of scalar fields mixed by turbulence. II, Spectral Theory. *Phys. Fluids* **11**, 2316–2327.

GIBSON, C. H., LYON, R. R. & HIRSCHSOHN, I. 1970 Reaction product fluctuations in a sphere wake. *AIAA J.* **8**, 1859–1863.

GRANT, H. L., HUGHES, B. A., VOGEL, W. M. & MOILLIET, A. 1968 The spectrum of temperature fluctuations in turbulent flow. *J. Fluid Mech.* **34**, 423–442.

GURVICH, A. S., KALLISTRATOVA, M. A. & MARTVEL, F. E. 1977 An investigation of strong fluctuations of light intensity in a turbulent medium at a small wave parameter. *Izv. Vyssh. Uchebn. Zaved. Radiofiz.* **20**, 1020–1031.

GURVICH, A. S., TIME, N. S., TUROVTSEVA, L. S. & TURCHIN, V. F. 1974 Restoring of the temperature fluctuation spectrum in the atmosphere from optical measurements. *Izv. Akad. Nauk SSSR, Atmos. Ocean. Phys.* **10**, 292–297.

HILL, R. J. 1978*a* Models of the scalar spectrum for turbulent advection. *J. Fluid Mech.* **88**, 541–562.

HILL, R. J. 1978*b* Optical propagation in turbulent water. *J. Opt. Soc. Am.* **68**, 1067–1072.

HILL, R. J. 1978*c* Spectra of fluctuations of refractivity, temperature, humidity, and the temperature-humidity cospectrum in the inertial and dissipation ranges. *Radio Sci.* **13**, 953–961.

HILL, R. J. 1980 Solution of Howells' model of the scalar spectrum and comparison with experiment. *J. Fluid Mech.* **96**, 705–722.

HILL, R. J. & CLIFFORD, S. F. 1978 Modified spectrum of atmospheric temperature fluctuations and its application to optical propagation. *J. Opt. Soc. Am.* **68**, 892–899.

KELLER, B. S. & YAGLOM, A. M. 1970 Effect of energy dissipation fluctuations on form of turbulent spectrum in extreme shortwave region. *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza* **5**, 70–79.

LESIEUR, M. & ROGALLO, R. 1989 Large-eddy simulation of passive scalar diffusion in isotropic turbulence. *Phys. Fluids* A **1**, 718–722.

Monin, A. S. & Yaglom, A. M. 1975 *Statistical Hydrodynamics*, vol. 2. M.I.T. Press.

Oboukhov, A. M. 1949 Structure of the temperature field in turbulent flow. *Izv. Akad. Nauk SSSR, Ser. Geogr. Geofiz.* **13**, 58–69.

O'Brien, E. E. & Francis, G. S. 1963 A consequence of the zero fourth cumulant approximation. *J. Fluid Mech.* **13**, 369–382.

Park, J. T. 1976 Inertial subrange turbulence measurements in the marine boundary layer. Ph.D thesis, University of California, San Diego.

Sarpkaya, T. 1979 Vortex-induced oscillations. A selective review. *J. Appl. Mech.* **46**, 241–258.

Tatarskii, V. I. 1956 Microstructure of the temperature field in the ground air layer. *Izv. Akad. Nauk SSSR, Ser. Geofiz.* **6**, 669–679.

Tatarskii, V. I. 1971 The effects of the turbulent atmosphere on wave propagation. US Dept. of Commerce, National Technical Information Service, Springfield, Va.

Time, N. S. 1981 Investigation of temperature microstructure with the aid of optical measurements in the atmosphere. *Izv. Akad. Nauk SSSR, Atmos. Ocean. Phys.* **17**, 117–121.

Williams, R. M. & Paulson, C. A. 1977 Microscale temperature and velocity spectra in the atmospheric boundary layer. *J. Fluid Mech.* **83**, 547–567.

Yaglom, A. M. 1949 Local structure of the temperature field in turbulent flow. *Dokl. Akad. Nauk SSSR* **69**, 743–746.

Yaglom, A. M. 1981 Laws of small-scale turbulence in atmosphere and ocean (in commemoration of the 40th anniversary of the theory of locally isotropic turbulence). *Izv. Akad. Nauk SSSR Atmos. Ocean. Phys.* **17**, 919–935.